Tracing pirate cards as part of the satellite video broadcasting

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REDOCS'16 Report

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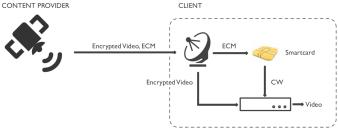


- 2 Performance Metrics
- 3 Strategy 1
- 4 Strategy 2
- 5 Strategy 3
- 6 Bilan & Perspective

# Table of contents

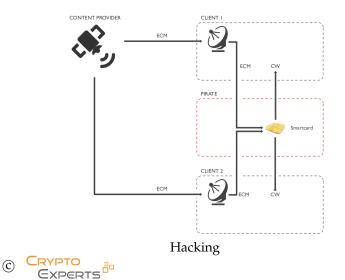
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- 3 Strategy 1
- 4 Strategy 2
- 5 Strategy 3
- 6 Bilan & Perspective

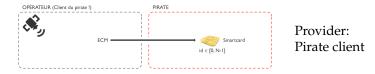
## Problem



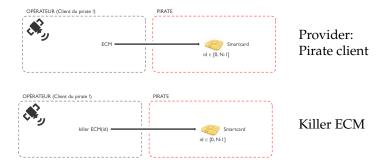
Satellite broadcasting





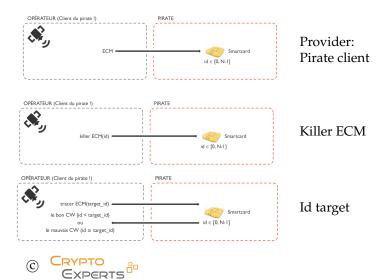








## Problem



## **Pirate Strategies**

1 strategy1(
$$r, CW_0, \ldots, CW_{n-1}$$
)  $\rightarrow CW_0$ 

2 strategy2(
$$r, CW_0, \ldots, CW_{n-1}$$
)  $\rightarrow \begin{cases} majority(CW_0, \ldots, CW_{n-1}) & \text{if } n \text{ is odd} \\ CW_0 & \text{else} \end{cases}$ 

**3** strategy3( $r, CW_0, \ldots, CW_{n-1}$ )  $\rightarrow CW_{r \mod n}$ 

# Table of contents

#### 1 Problem

#### 2 Performance Metrics

- 3 Strategy 1
- 4 Strategy 2
- 5 Strategy 3
- 6 Bilan & Perspective

## Metrics

- CPU Time (*s*)
- Collateral damage *ColD* (Avg, stddev):  $\sum (1 \frac{id_i}{N})$
- QoS of the pirate (Avg): 100 \* <sup>t</sup>/<sub>T</sub> (T: number of normal ECMs, t: number of correct cw)

# Table of contents

#### 1 Problem

#### **2** Performance Metrics

3 Strategy 1

- 4 Strategy 2
- 5 Strategy 3
- 6 Bilan & Perspective



# General principle

- Hypothesis: The traitor always uses the same card.
- Goal: Locate the card by using a minimum number of tracking ECM. Killing the card will
- Solution: Binary search (average number of iterations *log*(*n*))

## Benchmark

	CPU Time (s)	Collateral damage		QoS
		Avg	Stddev	Avg
Binary Search	54.37	14.11	8.14	0
Ternary Search	54.18	17.52	10.78	0

Table: Benchmark for 100 runs and nbCard = 10

# Table of contents

- **2** Performance Metrics
- 3 Strategy 1
- 4 Strategy 2
- 5 Strategy 3
- 6 Bilan & Perspective



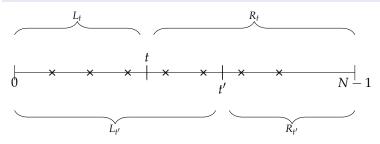
#### Notations

- $M_t$  : pirate response to ECM tracer t
- *M*<sub>t</sub> ∈ *L* : majority of pirate cards identifiers are < *t* (on the left side)
- $M_t \in R$  : majority of pirate cards identifiers are  $\geq t$  (on the right side)

## Strategy 2: Why binary search works.

#### Proposition

Let p = 2k + 1 cards (majority vote) and t' < t two tracers ECM.  $M_t \in L$  and  $M_{t'} \in R \implies \exists \mathbf{Id}_P \in [t', t[.$ 



# Algorithm

#### Pivots

$$p \leftarrow 0$$

$$p' \leftarrow N-1$$

#### Details

• Stops when |p - p'| = 1

■ ECM tracer 
$$t_m \leftarrow \lfloor (p + p')/2 \rfloor$$
  
if  $M_{t_m} \in L$ ,  $p' \leftarrow t_m$   
else  $p \leftarrow t_m$ 

# Benchmark Strategy II

	CPU Time (s)	Collateral damage		QoS
	CFU Time (s)	Avg	Stddev	Avg
Optimal approach	249.82	50.35	17.24	0
Paper approach [Tas05]	94.69	50.55	15.10	0

Table: Benchmark for 100 runs and nbCard = 10

Tamir Tassa, *Low bandwidth dynamic traitor tracing schemes*, J. Cryptol. **18** (2005), no. 2, 167–183.

# Table of contents

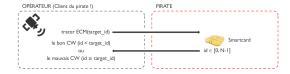
- **2** Performance Metrics
- 3 Strategy 1
- 4 Strategy 2
- 5 Strategy 3
- 6 Bilan & Perspective





#### Pirate Strategy

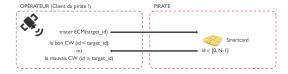
Own a number *n* of cards, and generates randomly and uniformly an number *r* larger than *n*.





#### Pirate Strategy

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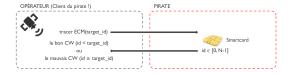


- made only of correct values cw



#### Pirate Strategy

Own a number *n* of cards, and generates randomly and uniformly an number *r* larger than *n*.

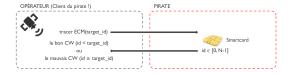


- made only of correct values cw
- made only of incorrect values of cw



#### Pirate Strategy

Own a number *n* of cards, and generates randomly and uniformly an number *r* larger than *n*.



- made only of correct values cw
- made only of incorrect values of cw
- made both of correct and incorrect values of cw



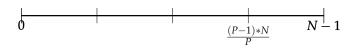
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Population : N-1 cards







#### P pirates cards $\rightarrow$ P - 1 intervals







Condition for dichotomy

In [A;B], if *NbrCardsFalse* > 0, then at least a pirate card is present.







With m intervals.



$$\frac{(P-m)*N}{P} \qquad \qquad N-1$$

#### Condition for dichotomy

In [A;B], if *NbreCardsFalse* - (nbCardsMute+0.6)\*Cst > 0, then at least a pirate card is present.

# Benchmark Strategy III

# CPU Time (s)Collateral damage<br/>AvgQoS<br/>StddevHeuristic approach631.29688653563057

Table: Benchmark for 100 runs and nbCard = 10

# General principle for Strategy III version II

- Return a set of small intervals that have a good probability to contain id of the traitor's cards
- Let  $S = \{0, 1, 2, ..., n 1\}$  be the set of all the cards (regular user and traitors).
- Divide *S* in 100 subsets and select subsets *S*<sub>*j*</sub> that pass the test.
- The test take a subset Sj = [a, b], uses a and b as input for a tracking ECM send nbSample times.
- Let *ProbA* be the chance to have negative response with tracking ECM *a* and *ProbB* be the chance to have positive response with tracking ECM *b*.
- Reject  $S_i$  if abs(ProbA ProbB) > epsilon and accept  $S_j$  otherwise.
- Repeat until  $|S_i| \leq 1000$ .

# Table of contents

- **2** Performance Metrics
- 3 Strategy 1
- 4 Strategy 2
- 5 Strategy 3
- 6 Bilan & Perspective



## Bilan

- Optimal counter attack against strategy I and II
- 2 heuristic approaches for strategy III



## Perspectives

- Find theoretical bound for strategy III
- Explore game theory alternative (Bilevel optimization)

$$\min_{id_i} \sum_{i} (1 - \frac{id_i}{N})$$
s.t.  $QoS(pirate) \le \epsilon$   
 $id_i \in \{0, 1\}$ 

### Bonus



#### Interesting tools and methods

Teamwork with efficiency (AGILE method)

■ Tools: GitHub, CollabEdit

#### Social

Contacts, colleagues, friends, fun...