Private Comparison

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Rencontres Entreprises DOCtorants Sécurité 2018

Scenario

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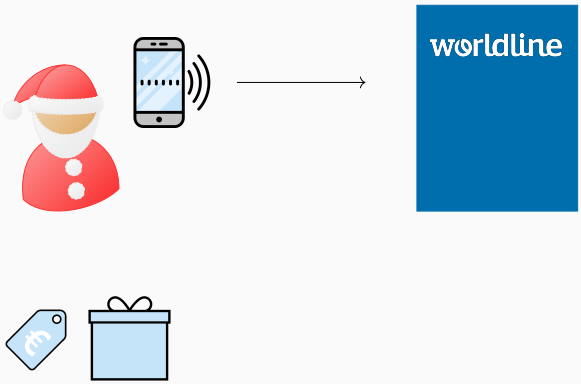

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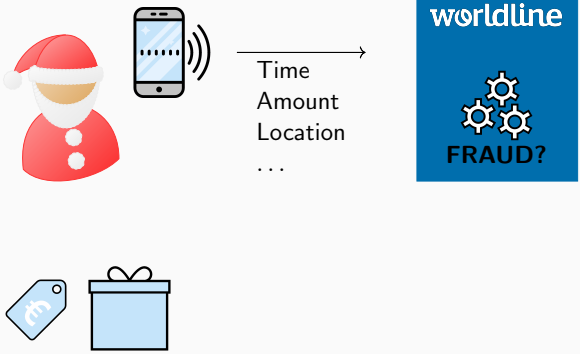

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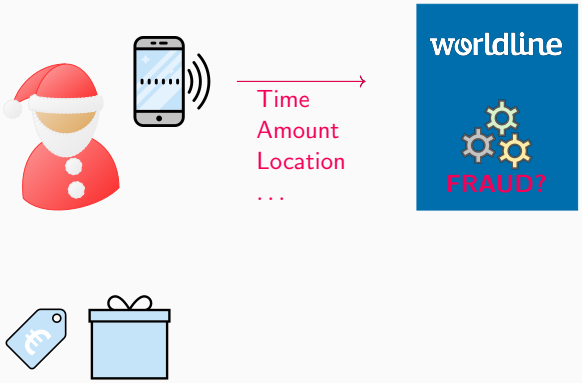


Goal: Detect **FRAUD** in $< 200ms$

Scheme

Private input: $x_1, \ldots, x_n$

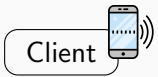
Server

worldline
e-payment services

Private inputs:

- Indv. thresholds: $t_1, \ldots, t_n$
- Weights: $\alpha_1, \ldots, \alpha_n$
- Total threshold: T

Scheme



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Interactive Protocol

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Output: $[\sum_{i=1}^{n} \alpha_i [x_i > t_i] > T]$

$[x > t] = 1$ if $x > t$ and 0 otherwise

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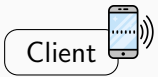

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Interactive Protocol

Output: $[\sum_{i=1}^{n} \alpha_i [x_i > t_i] > T]$

$[x > t] = 1$ if $x > t$ and 0 otherwise

Server only learns if a fraud occurred

Client learns nothing

Security Model

Sec. w.r.t. Server

Server does not learn:

- individual comparisons
- individual x_i's
- sum of weighed comparisons

Sec. w.r.t. Client

Client does not learn:

- weights α_i
- individual thresholds t_i
- total threshold T

Assumption on Client

Client is honest but curious:

- Follows the protocol honestly
- Tries to learn more than his inputs

Secure against honest but curious:

- Learns nothing beyond his inputs

Outline

Unsuccessful Ideas

Garbled Circuits

Idea: large garbled circuit containing sub circuits for individual comparisons.

Garbled Circuits

Idea: large garbled circuit containing sub circuits for individual comparisons.

+ Secure equality test or comparison
+ Communication-efficient
− Too slow!

Fully Homomorphic Encryption

Properties

- Absorption : $\mathsf{E}(x) * \alpha = \mathsf{E}(x * \alpha)$
- Addition : $\mathsf{E}(x) + \mathsf{E}(y) = \mathsf{E}(x + y)$
- Multiplication : $\mathsf{E}(x) * \mathsf{E}(y) = \mathsf{E}(x * y)$

Security

- Indistinguishable ciphertexts

PIR : Private Information Retrieval

$PIR(x, D)$

- The client sends $Q = (\mathrm{E}(q_0), ..., \mathrm{E}(q_l))$
- $q_x = 1$, $q_{k \neq x} = 0$
- The server computes the PIR with $D = (d_0, ..., d_l)$

PIR : Private Information Retrieval

$$
\begin{aligned}
d_1 \times \boxed{\mathrm{E}(0)} &= \boxed{\mathrm{E}(0)} \\
d_2 \times \boxed{\mathrm{E}(0)} &= \boxed{\mathrm{E}(0)} \quad + \\
d_3 \times \boxed{\mathrm{E}(1)} &= \boxed{\mathrm{E}(d_3)} \quad + \\
d_4 \times \boxed{\mathrm{E}(0)} &= \boxed{\mathrm{E}(0)} \quad + \quad \rightarrow \boxed{\mathrm{E}(0+0+d_3+0+0+0)} \\
d_5 \times \boxed{\mathrm{E}(0)} &= \boxed{\mathrm{E}(0)} \quad + \\
d_6 \times \boxed{\mathrm{E}(0)} &= \boxed{\mathrm{E}(0)} \quad +
\end{aligned}
$$

Three-party Solution

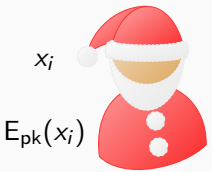
worldline

t_i, α_i

T

sk, pk

Three-party Solution



$$n\times\ \mathsf{PIR}(x_i, b_i)$$

worldline

t_i, α_i

T

$$b_i = \left(b_i^j\right)_{1\leq j\leq 2^{|x_i|}} = \begin{cases} 0 & \text{if } j < t_i \\ \alpha_i & \text{otherwise} \end{cases}$$

sk, pk

Three-party Solution

x_i

$\mathsf{E}_{\mathsf{pk}}(x_i)$

$n\times$ PIR(x_i, b_i)

worldline

t_i, α_i

T

$$C := \mathsf{E}_{\mathsf{pk}}\left(\sum_{i=1}^{n} \alpha_i[x_i > t_i] - T\right)$$

$$b_i = \left(b_i^j\right)_{1\le j\le 2^{|x_i|}} = \begin{cases} 0 & \text{if } j < t_i \\ \alpha_i & \text{otherwise} \end{cases}$$

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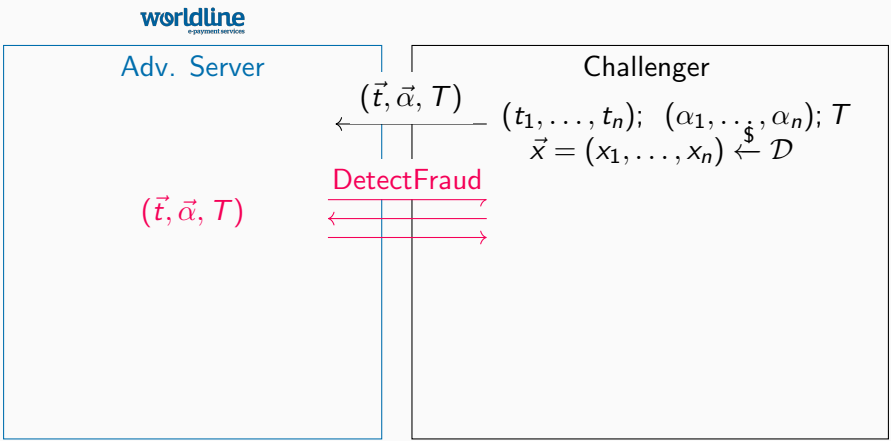
Retained Solutions

Security with Respect to the Server



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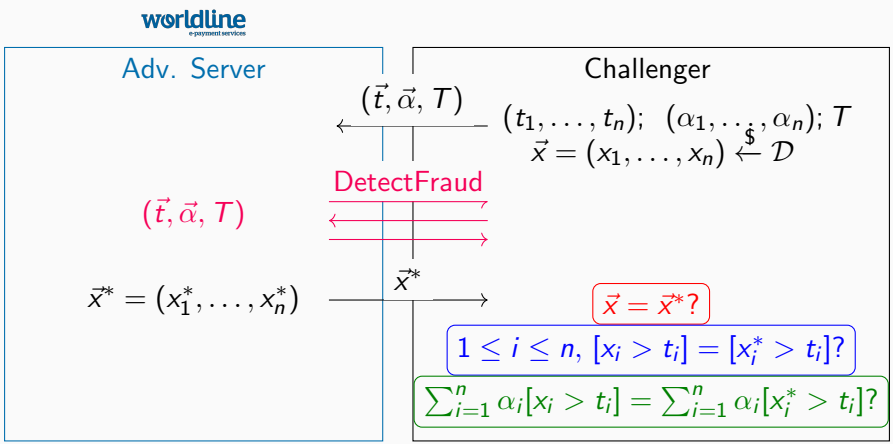
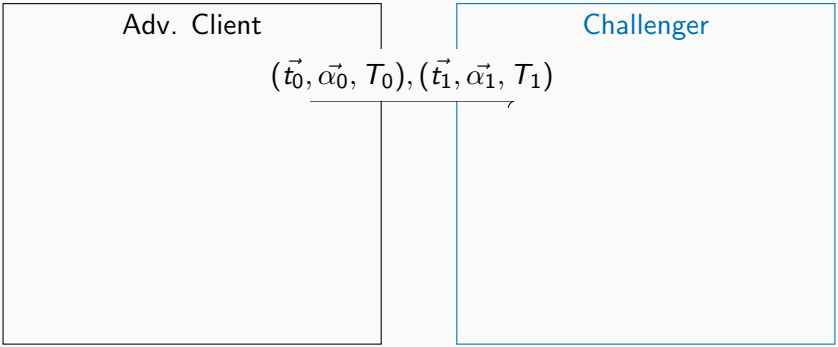
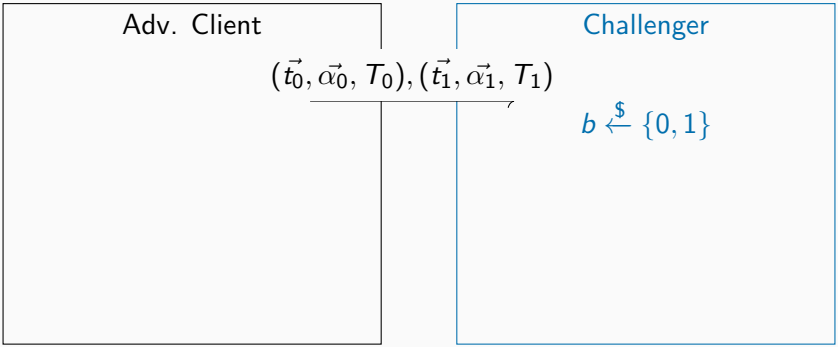


Security with Respect to the Client



Security with Respect to the Client



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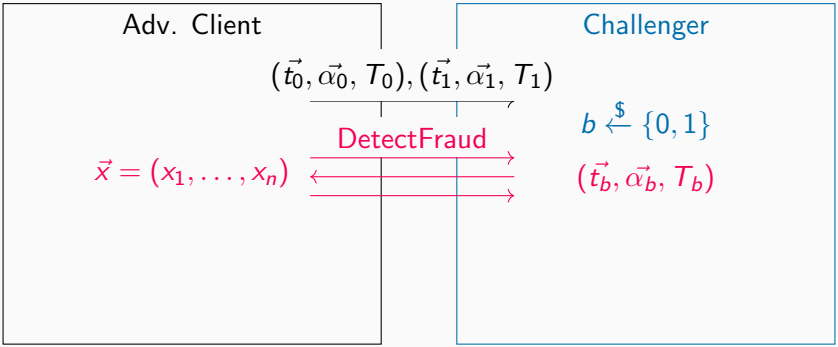

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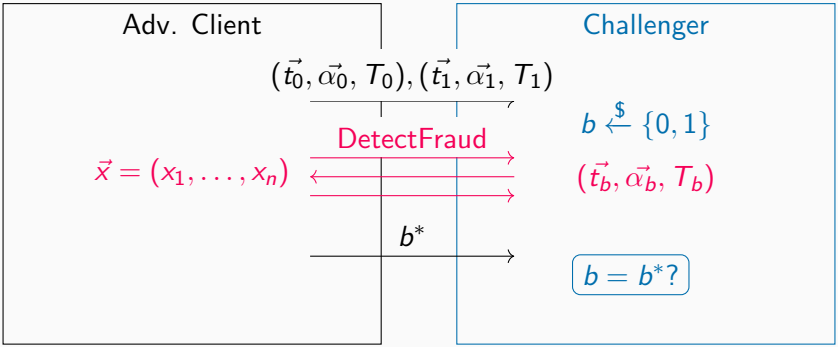

Somewhat Homomorphic Encryption

Ring Learning With Error

- Noise hides secret
- Noise growth : Addition $=$, Absorption $+$, Multiplication $++$

Example

- Secret : p, large random : q, small random r
- Encrypt binary m : $c = q \cdot p + 2 \cdot r + m$
- Decrypt : $m = (c \mod p) \mod 2$

Example

- If $r > p$ decryption fails
- Addition : $c_1 + c_2 = p \cdot (q_1 + q_2) + 2 \cdot (r_1 + r_2) + m_1 + m_2$
- Multiplication :
  $c_1 * c_2 = p \cdot (c_2 q_1 + c_1 q_2 - q_1 q_2) + 2 \cdot (2 r_1 r_2 + m_1 r_2 + m_2 r_1) + m_1 \cdot m_2$

Parameters

- Plaintext Modulus : p
- Ciphertext Modulus : q
- Ring : $\mathbb{Z}_p[X]/(X^n + 1)$

Link

- Security : $+$ when $n+$, $-$ when $q+$
- Batching : $m_2X^2 + m_1X^1 + m_0$
- p prime, $p = 1 \mod (2n)$

Koda

$x_1, \ldots, x_n$

worldline
e-payment services

Server pk, sk

$t_1, \ldots, t_n, \alpha_1, \ldots, \alpha_n, T$

Koda

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$$
\begin{array}{l}
\mathsf{E}_{\mathsf{pk}}(0) \\
\cdots \\
\mathsf{E}_{\mathsf{pk}}(0) \\
\mathsf{E}_{\mathsf{pk}}(\alpha_i) \quad \leftarrow t_i, i \in \{1, \cdots, n\} \\
\cdots \\
\mathsf{E}_{\mathsf{pk}}(\alpha_i) \\
C_T \coloneqq \mathsf{E}_{\mathsf{pk}}(T)
\end{array}
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Koda

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$\longleftarrow$

For all i, select the x_ith
$C_i \coloneqq \mathsf{E}_{\mathsf{pk}}(\alpha_i[x_i > t_i])$
$r > 0, \quad C = (\sum_{i=1}^{n} C_i - C_T) \cdot r$

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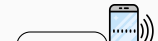

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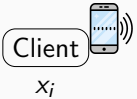
$\mathsf{D}_{\mathsf{sk}}(\lceil C \rfloor)$

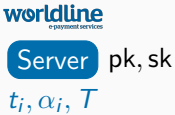
$$C = \mathsf{E}_{\mathsf{pk}}([\textstyle\sum_{i=1}^{n} \alpha_i[x_i > t_i] > T] \cdot r)$$

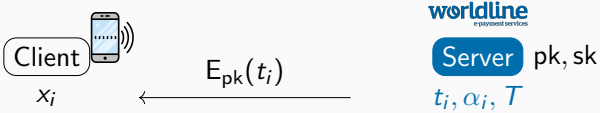
Tricks with this solution

- Easy to make ANDs of Comparison
- For one x_i, multiple α_i

Kenaï



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$\mathsf{E}_{\mathsf{pk}}(t_i)$
worldline
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pk, sk
t_i, α_i, T

Kenaï

$\mathsf{E}_{\mathsf{pk}}(t_i)$ ⟵

x_i

$b_i \xleftarrow{\$} \{-1, 1\}$

R_i, r_i random

$R_i > r_i > 0$

$c_i = R_i \cdot b_i \cdot (\mathsf{E}_{\mathsf{pk}}(t_i) - \mathsf{E}_{\mathsf{pk}}(x_i)) - \mathsf{E}_{\mathsf{pk}}(r_i)$

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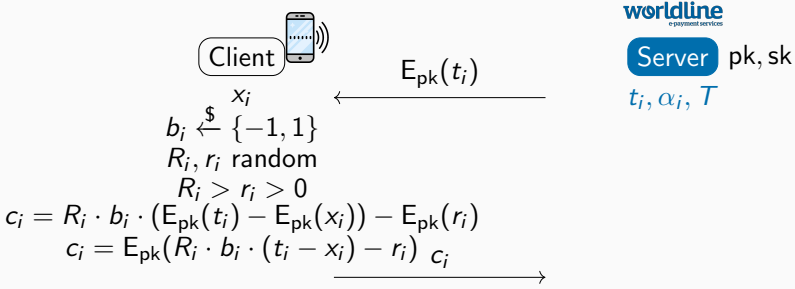
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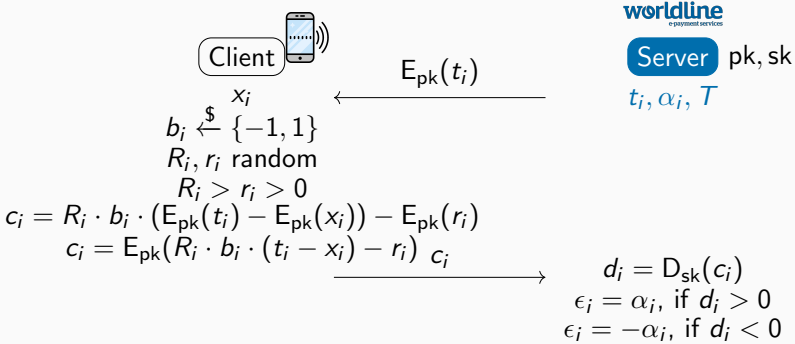
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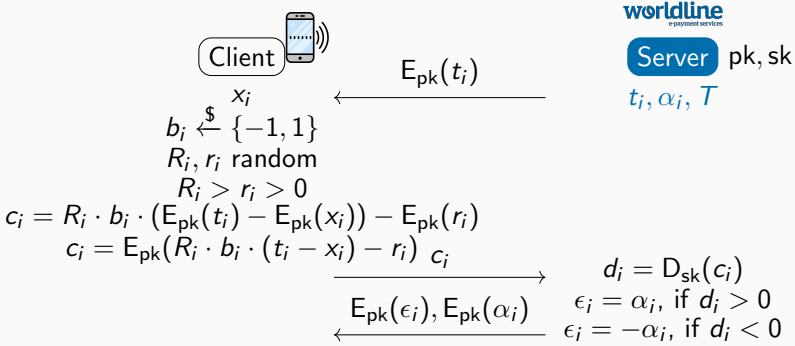
Kenaï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Kenaï

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← $\mathsf{E}_{\mathsf{pk}}(\epsilon_i), \mathsf{E}_{\mathsf{pk}}(\alpha_i)$

$$b_i \cdot \mathsf{E}_{\mathsf{pk}}(\epsilon_i) = \pm \mathsf{E}_{\mathsf{pk}}(\alpha_i)$$

$$a_i = b_i \cdot \mathsf{E}_{\mathsf{pk}}(\epsilon_i) + \mathsf{E}_{\mathsf{pk}}(\alpha_i) \in \{\mathsf{E}_{\mathsf{pk}}(0), \mathsf{E}_{\mathsf{pk}}(2\alpha_i)\}$$

$$U = \sum_{i=1}^{n} a_i = \mathsf{E}_{\mathsf{pk}}(2S)$$

$$S = \sum_{i=1}^{n} \alpha_i \cdot [x_i > t_i]$$

Kenaï

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Implementation

Software requirements

- SEAL 3.0
- C++

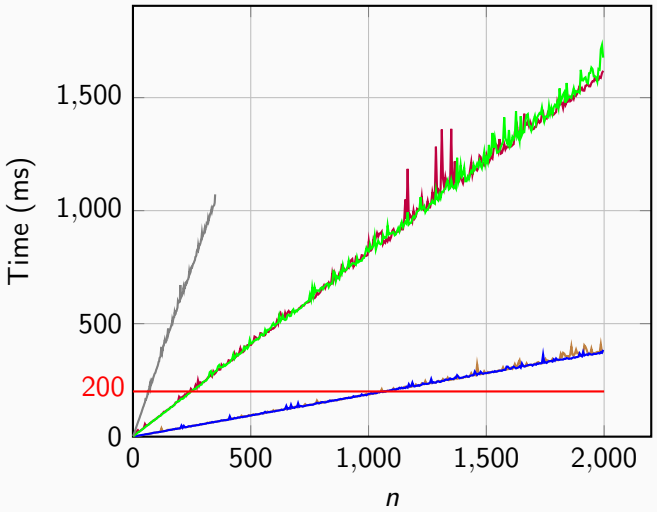
Encryption parameters

- Encryption scheme: FV-RNS variant (BFV)
- Security: 128 bits

Benchmark platform

- Intel® Core™ i7-6500U CPU @ 2.50GHz

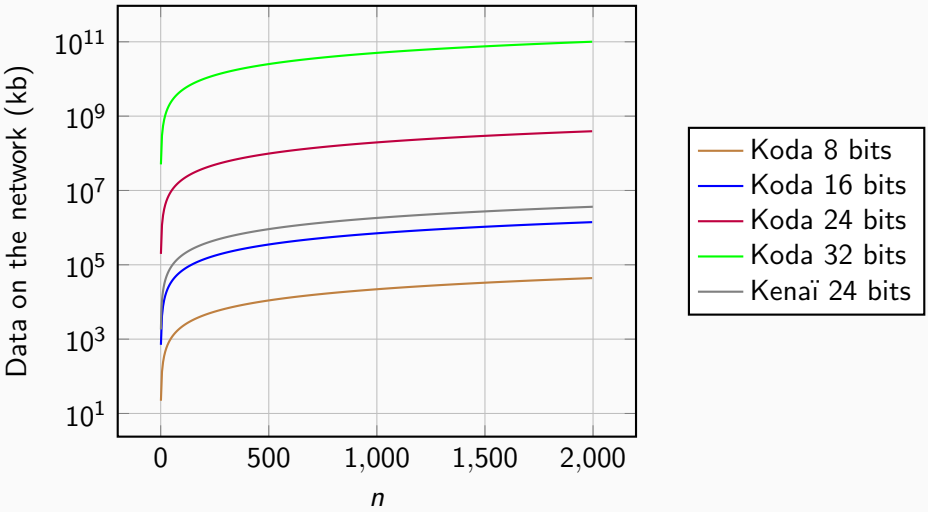
Koda & Kenaï:



- Koda 8 & 16 bits: $n \approx 1050$
- Koda 24 & 32: $n \approx 250$
- Kenaï 24 bits: $n \approx 65$

Koda 8 bits
Koda 16 bits
Koda 24 bits
Koda 32 bits
Kenaï 24 bits
Maximum computation time

Simulated network cost



Recap

	Koda				Kenaï
word size (bits)	8	16	24	32	24
n max	1050		250		65
cost @ n max (MB)	23	740	4800	12381600	118

Limitations

Evaluating our Solution

Pros:

- Client: little memory
- Server: fast

Cons:

- Malicious Client can cheat
- Information leak on Client inputs (one time security)
- Client side computations expensive for payment terminal

Conclusion

Conclusion

Results

- Koda and Kenaï allow a lot of comparisons

Perspective

- Improve the network costs
- Resist against a malicious client

Thank You

- Questions ?